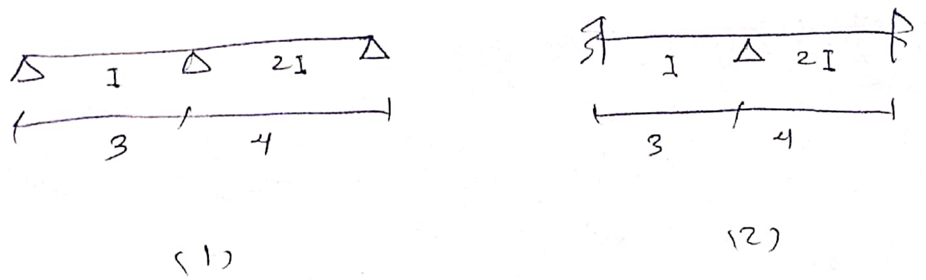
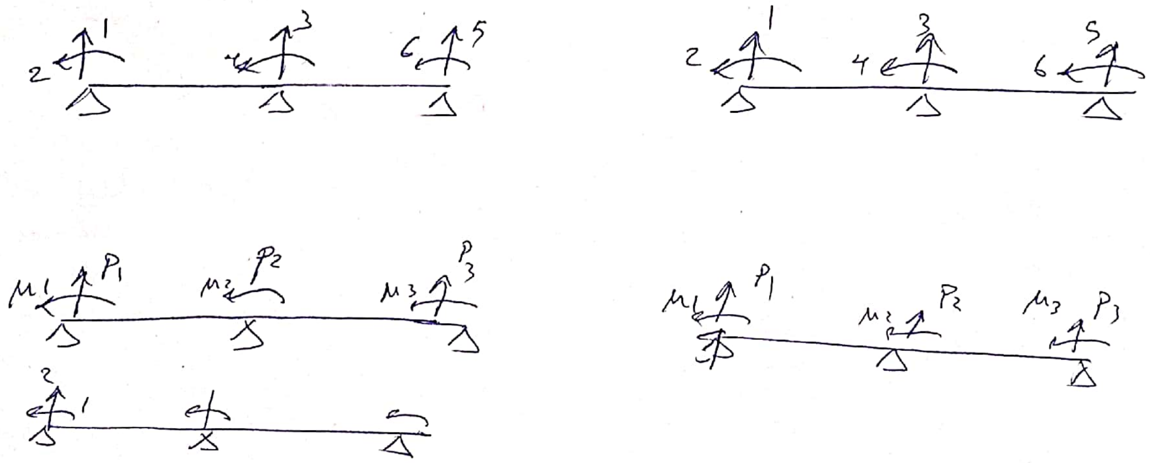
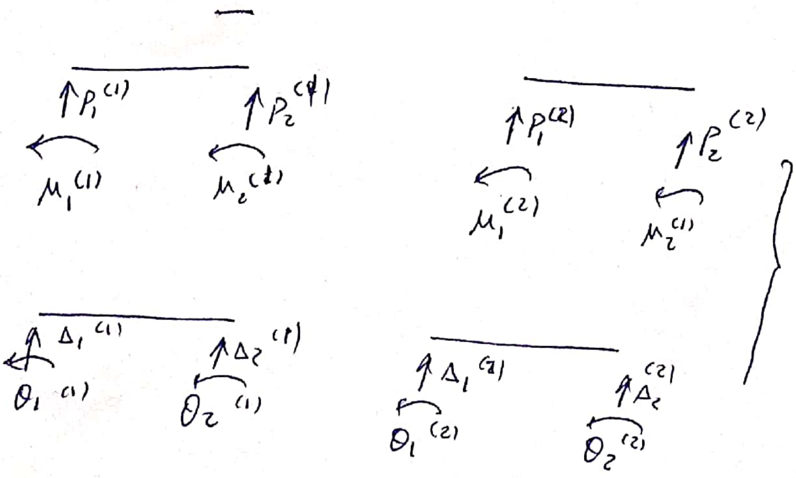
شکل ←


I
2I
3
4
(1)
(2)


1
2
3


M_1 P_1
M_2 P_2
M_3 P_3

[illegible]


$P_1^{(1)}$ $P_2^{(1)}$
$M_1^{(1)}$ $M_2^{(1)}$
$P_1^{(2)}$ $P_2^{(2)}$
$M_1^{(2)}$ $M_2^{(2)}$
$\Delta_1^{(1)}$ $\Delta_2^{(1)}$
$\theta_1^{(1)}$ $\theta_2^{(1)}$
$\Delta_1^{(2)}$ $\Delta_2^{(2)}$
$\theta_1^{(2)}$ $\theta_2^{(2)}$

عضو

$$K^{(e)} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & \text{SYM} & & \frac{4EI}{L} \end{bmatrix}$$

$$K^{(1)} = \begin{bmatrix} \frac{12}{27}EI & \frac{6}{9}EI & \frac{-12}{27}EI & \frac{6}{9}EI \\ & \frac{4}{3}EI & \frac{-6}{9}EI & \frac{2}{3}EI \\ & & \frac{12}{27}EI & \frac{-6EI}{9} \\ & & & \frac{4EI}{3} \end{bmatrix}$$

$$K^{(1)} = \frac{EI}{27} \begin{bmatrix} 12 & 18 & -12 & 18 \\ 18 & 36 & -18 & 18 \\ -12 & -18 & 12 & -18 \\ 18 & \underset{18}{\text{SYM}} & -18 & 36 \end{bmatrix}$$

$$K^{(2)} = \begin{bmatrix} \frac{12\times 2EI}{64} & \frac{6\times 2EI}{16} & \frac{-12EI\times 2}{64} & \frac{6\times 2EI}{16} \\ & \frac{4\times 2EI}{4} & -\frac{6\times 2EI}{16} & \frac{2EI\times 2}{4} \\ & & \frac{12\times 2EI}{64} & \frac{-6\times 2EI}{16} \\ & & & \frac{4\times 2EI}{4} \end{bmatrix}$$

$$K^{(2)} = \frac{EI}{8} \begin{bmatrix} 3 & 6 & -3 & 6 \\ 6 & 16 & -6 & 8 \\ -3 & -6 & 3 & -6 \\ 6 & 8 & -6 & 16 \end{bmatrix}$$

درجات آزادی اعضا

جدول کد

	1	2	3	4
اعضا 1	1	2	3	4
2	3	4	5	6

$K_{11} = K_{11}^{(1)} = \frac{12}{27} EI$

$K_{12} = K_{12}^{(1)} = \frac{18}{27} EI$

$K_{13} = K_{13}^{(1)} = \frac{-12}{27} EI$

$K_{14} = K_{14}^{(1)} = \frac{18}{27} EI$

$K_{15} = K_{16} = 0$

$K_{22} = K_{22}^{(1)} = \frac{36}{27} EI$

$K_{23} = K_{23}^{(1)} = -\frac{18}{27} EI$

$K_{24} = K_{24}^{(1)} = \frac{18}{27} EI$

$K_{25} = K_{26} = 0$

$K_{33} = K_{33}^{(1)} + K_{11}^{(1)} = \frac{12}{27} EI + \frac{3EI}{8} = \frac{177}{216} EI = \frac{59}{72} EI$

$K_{34} = K_{34}^{(1)} + K_{12}^{(1)} = \frac{-18EI}{27} + \frac{6EI}{8} = \frac{18}{216} EI = \frac{6}{72} EI$

$K_{35} = K_{13}^{(2)} = -\frac{3EI}{8}$

$K_{36} = K_{14}^{(2)} = \frac{6}{8} EI$

$K_{44} = K_{44}^{(1)} + K_{22}^{(2)} = \frac{36}{27} EI + \frac{16}{8} EI = \frac{240}{72} EI$

$K_{45} = K_{23}^{(2)} = -\frac{6EI}{8}$

$K_{46} = K_{24}^{(2)} = \frac{8EI}{8}$

$K_{55} = K_{33}^{(2)} = \frac{3EI}{8}$

$K_{56} = -\frac{6EI}{8}$

$K_{66} = \frac{16}{8} EI$

$$K = EI \begin{bmatrix} \frac{12}{27} & \frac{18}{27} & -\frac{12}{27} & \frac{18}{27} & 0 & 0 \\ & \frac{36}{27} & -\frac{18}{27} & \frac{18}{27} & 0 & 0 \\ & & \frac{59}{72} & \frac{6}{72} & -\frac{3}{8} & \frac{6}{8} \\ & & & \frac{240}{72} & -\frac{6}{8} & 1 \\ & & & & \frac{3}{8} & -\frac{6}{8} \\ & & & & & 2 \end{bmatrix}$$

$$K = \frac{EI}{72} \begin{bmatrix} 32 & 48 & -32 & 48 & 0 & 0 \\ 48 & 96 & -48 & 48 & 0 & 0 \\ -32 & -48 & 59 & 6 & -27 & 54 \\ 48 & 48 & 6 & 240 & -\cancel{27}\,54 & 72 \\ 0 & 0 & -27 & -54 & 27 & -54 \\ 0 & 0 & 54 & 72 & -54 & 144 \end{bmatrix}$$

ماتریس سختی کل سیستم برای هر دو سیستم ۱ و ۲ یکسان است؛

تنها تفاوت دو سیستم در شرایط تکیه‌گاهی است؛

$$\underset{\sim}{F} = \underset{\sim}{K}\,\underset{\sim}{\Delta}$$

سیستم ۱ و ۲ :

سیستم (1)

$$\Delta = \begin{Bmatrix} \Delta_1 = 0 \\ \theta_1 \\ \hline \Delta_2 = 0 \\ \theta_2 \\ \hline \Delta_3 = 0 \\ \theta_3 \end{Bmatrix}$$

سه مجهول دارد

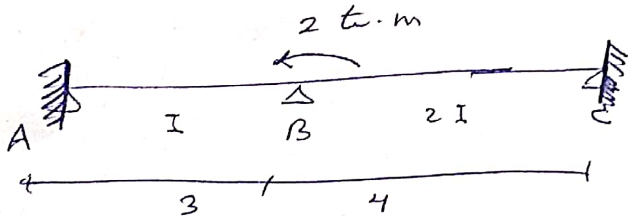
$\theta_1, \theta_2, \theta_3$

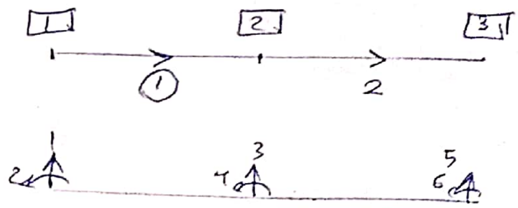
سیستم (2)

$$\Delta = \begin{Bmatrix} \Delta_1 = 0 \\ \theta_1 = 0 \\ \hline \Delta_2 = 0 \\ \theta_2 \\ \hline \Delta_3 = 0 \\ \theta_3 = 0 \end{Bmatrix}$$

یک مجهول دارد θ_2

مثال: مطلوب است محاسبه‌ی نیروها و تغییرشکل‌ها برای سازه زیر!





$$\begin{Bmatrix} P_1 \\ M_1 \\ \hline P_2 \\ M_2 \\ \hline P_3 \\ M_3 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ M_1 \\ \hline P_2 \\ M_2 = 2 \\ \hline P_3 \\ M_3 \end{Bmatrix} \begin{matrix} \rightarrow \text{تکیه‌گاه} \\ \\ \rightarrow \text{〃} \\ \\ \rightarrow \text{تکیه‌گاه} \end{matrix}$$

بارهایی وارد بر گره‌ها

$$\begin{Bmatrix} \Delta_1 = 0 \\ \theta_1 = 0 \\ \Delta_2 = 0 \\ \theta_2 \\ \Delta_3 = 0 \\ \theta_3 = 0 \end{Bmatrix} \rightarrow \text{مجهول}$$

$$\begin{Bmatrix} P_1 \\ M_1 \\ P_2 \\ M_2 \\ P_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{16} \\ & K_{22} & \cdots & K_{26} \\ & & K_{33} \cdots & K_{36} \\ & & K_{44} \cdots & K_{46} \\ & & K_{55} & K_{56} \\ & & & K_{66} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \hline 0 \\ \theta_2 \\ \hline 0 \\ 0 \end{Bmatrix}$$

$P_1 = k_{14}\ \theta_2$ $\quad P_2 = k_{34}\ \theta_2$ $\quad P_3 = k_5\ \theta_2$

$M_1 = k_{24}\ \theta_2$ $\quad M_2 = k_{44}\ \theta_2$ $\quad M_3 = k_{64}\ \theta_2$.

$\longrightarrow$ $M_2 = 2$ t.m $\longrightarrow$ $\theta_2 = \dfrac{M_2}{k_{44}} = \dfrac{2}{\frac{240 \times 1000}{72}} = 6 \times 10^{-4}$ (rad)

$k_{44} = \dfrac{240\ EI}{72}$

$EI = 10^{10}\ kg.cm^2 = 1000\ t.m^2$

$P_1 = \left(\dfrac{48}{72} \times 1000\right) \times 6 \times 10^{-4} = 0.4$ t

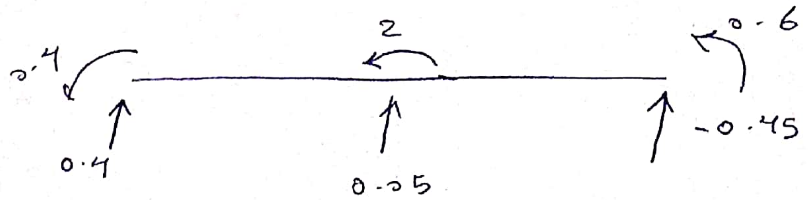
$M_1 = \left(\dfrac{48}{72} \times 1000\right) \times 6 \times 10^{-4} = 0.4$ t.m

$P_2 = \dfrac{6EI}{72}\theta_2 = \dfrac{6 \times 1000}{72} \times 6 \times 10^{-4} = 0.05$ t.m

$M_2 = 2$ t.m

$P_3 = \dfrac{-54}{72} \times 1000 \times 6 \times 10^{-4} = -0.45$ t

$M_3 = \dfrac{72}{72} \times 1000 \times 6 \times 10^{-4} = 0.6$ t.m



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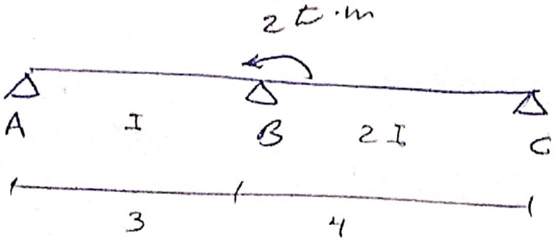
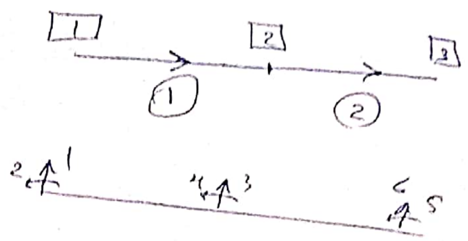
$\Sigma F_y = 0.4 + 0.05 - 0.45 = 0$ ✓

$\Sigma M_{A} = 0.4 + 2 + 0.6 + 0.05 \times 3 + (-0.45 \times 7) = 0$ ✓





$$\begin{Bmatrix} P_1 \\ M_1 \\ \hline P_2 \\ M_2 \\ \hline P_3 \\ M_3 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ 0 \\ \hline P_2 \\ 2 \\ \hline P_3 \\ 0 \end{Bmatrix} \qquad \begin{Bmatrix} \Delta_1 = 0 \\ \theta_1 \\ \hline \Delta_2 = 0 \\ \theta_2 \\ \hline \Delta_3 = 0 \\ \theta_3 \end{Bmatrix}$$

بيكون عارفهم داخله $\begin{Bmatrix} M_1 \\ M_2 \\ M_3 \end{Bmatrix}$, $\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix}$

لـ عكس الحل

مجهولات $\theta_1, \theta_2, \theta_3$

$$\begin{Bmatrix} P_1 \\ M_1 \\ P_2 \\ M_2 \\ P_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ & k_{32} & & & & \\ & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ & & & & & \\ & & & & & k_{66} \end{bmatrix} \begin{Bmatrix} 0 \\ \theta_1 \\ \hline 0 \\ \theta_2 \\ \hline 0 \\ \theta_3 \end{Bmatrix}$$

$$M_1 = k_{22}\theta_1 + k_{24}\theta_2 + k_{26}\theta_3 = 0$$

$$M_2 = K_{42}\theta_1 + K_{44}\theta_2 + K_{46}\theta_3 = 2$$

$$M_3 = K_{62}\theta_1 + K_{64}\theta_2 + K_{66}\theta_3 = 0$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{bmatrix} k_{22} & k_{24} & k_{26} \\ & k_{44} & k_{46} \\ & & k_{66} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix}$$

$$\begin{Bmatrix} M_1 = 0 \\ M_2 = 2 \\ M_3 = 0 \end{Bmatrix} = \frac{EI}{72} \begin{bmatrix} 96 & 48 & 0 \\ & 240 & 72 \\ & & 144 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix}$$

$$\underset{\sim}{F} = \underset{\sim}{K}\,\underset{\sim}{\Delta} \longrightarrow \underset{\sim}{\Delta} = \underset{\sim}{K}^{-1}\,\underset{\sim}{F}$$

$$\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} -4 \\ 8 \\ -4 \end{Bmatrix} \times 10^{-4}$$

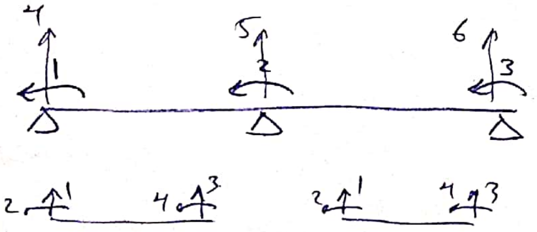
$$\begin{Bmatrix} P_1 \\ M_1 \\ \hline P_2 \\ M_2 \\ \hline P_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix} \begin{Bmatrix} 0 \\ -4 \\ \hline 0 \\ 8 \\ \hline 0 \\ -4 \end{Bmatrix} \times 10^{-4}$$

P_1, P_2, P_3 تعیین می‌شوند

راه حل دوم ←

ترتیب شماره گذاری درجات آزادی:

ابتدا درجات آزادی فعال و سپس درجات آزادی غیرفعال (محدود شده)



	1	2	3	4
1	4	1	5	2
2	5	2	6	3

2k

$$K_{11} = K_{22}^{(1)} = \frac{4}{3} EI$$
$$K_{12} = K_{24}^{(1)} = \frac{2}{3} EI$$
$$K_{13} = 0 = 0$$
$$K_{14} = K_{21}^{(1)} = \frac{6}{9} EI$$
$$K_{15} = 0 = 0$$
$$K_{16} = 0 = 0$$

$$K_{22} = K_{44}^{(1)} + K_{22}^{(2)}$$
$$K_{23} = 0$$
$$K_{24} = K_{41}^{(1)}$$
$$K_{25} = K_{43}^{(1)} + K_{21}^{(2)}$$
$$K_{26} = K_{23}^{(2)}$$

$$K_{33} = K_{44}^{(2)}$$
$$K_{34} = 0$$
$$K_{35} = K_{41}^{(2)}$$
$$K_{36} = K_{43}^{(2)}$$

$$K_{44} = K_{11}^{(1)}$$
$$K_{45} = K_{13}^{(1)}$$
$$K_{46} = 0$$

$$K_{55} = K_{33}^{(1)} + K_{11}^{(2)}$$
$$K_{56} = K_{13}^{(2)}$$

$$K_{66} = K_{33}^{(2)}$$

$$\begin{Bmatrix} M_1 = 0 \\ M_2 = 2 \\ M_3 = 0 \\ P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \frac{EI}{72} \left[\begin{array}{ccc|ccc} 96 & 48 & 0 & 48 & -48 & 0 \\ & 240 & 72 & 48 & 6 & -54 \\ & & 144 & 0 & 54 & -54 \\ \hline & & & 32 & -32 & 0 \\ & & & & 54 & -27 \\ & & & & & 27 \end{array}\right] \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \hline \Delta_1 = 0 \\ \Delta_2 = 0 \\ \Delta_3 = 0 \end{Bmatrix}$$

$$\left[\begin{array}{c|c} K_{uu} & K_{uc} \\ \hline K_{cu} & K_{cc} \end{array}\right] \begin{Bmatrix} \underset{\sim}{\theta} \\ \underset{\sim}{\Delta} \end{Bmatrix}$$

$$\begin{Bmatrix} F_u \\ F_c \end{Bmatrix} = \begin{bmatrix} K_{uu} & K_{uc} \\ K_{cu} & K_{cc} \end{bmatrix} \begin{Bmatrix} \underset{\sim}{\Delta_u} \\ \underset{\sim}{\Delta_c} \end{Bmatrix}$$

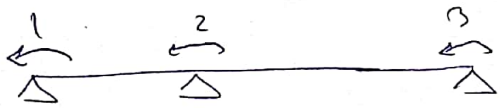
$$F_u = K_{uu}\,\Delta_u + K_{uc}\,\cancel{\Delta_c}^{\,0}$$

$$\boxed{F_u = K_{uu}\,\Delta_u}$$

$$F_c = K_{cu}\,\Delta_u + K_{cc}\,\cancel{\Delta_c}^{\,0}$$

$$\boxed{F_c = K_{cu}\,\Delta_u}$$

روش سوم ← روش تفکیک درجات آزادی مقید در غیر سیستم



	1	2	3	4
1	0	1	0	2
2	0	2	0	3

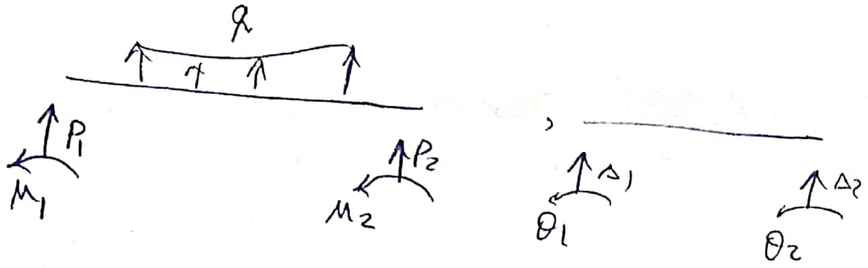
$$\begin{cases} K_{11} = K_{22}^{(1)} \\ K_{12} = K_{24}^{(1)} \\ K_{13} = 0 \\ \cancel{K_{14} = 0} \end{cases} \qquad \begin{cases} K_{22} = K_{44}^{(1)} + K_{22}^{(2)} \\ K_{23} = K_{24}^{(2)} \\ \cancel{K_{24} = } \end{cases} \qquad \begin{cases} K_{33} = K_{44}^{(2)} \end{cases}$$

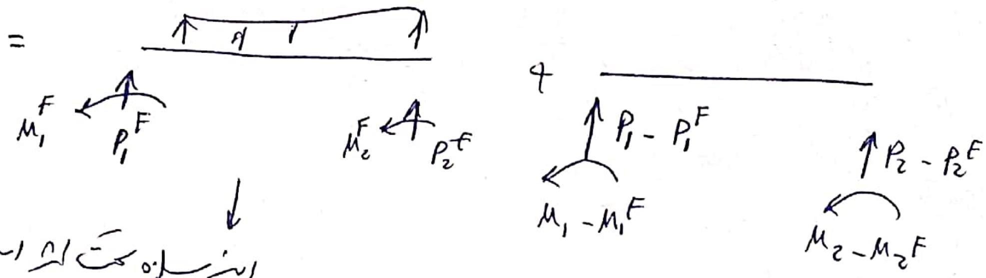
$$K_{u} = \underline{K} = \begin{bmatrix} K_{22}^{(1)} & K_{24}^{(1)} + \cancel{K_{22}^{(2)}} & 0 \\ & K_{44}^{(1)} + K_{22}^{(2)} & K_{24}^{(2)} \\ & & K_{44}^{(2)} \end{bmatrix}$$

$$\underset{\sim}{F} = \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2 \\ 0 \end{Bmatrix}$$

$$\Delta = \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} \quad — \quad \underset{\sim}{F} = \underset{\sim}{K}\, \underset{\sim}{\Delta}$$

بارها در طول عضو ؟





این مسئله تحت اثر این بارگذاری

$\Delta_1 = 0$
$\theta_1 = 0$

$\Delta_2 = 0$
$\theta_2 = 0$

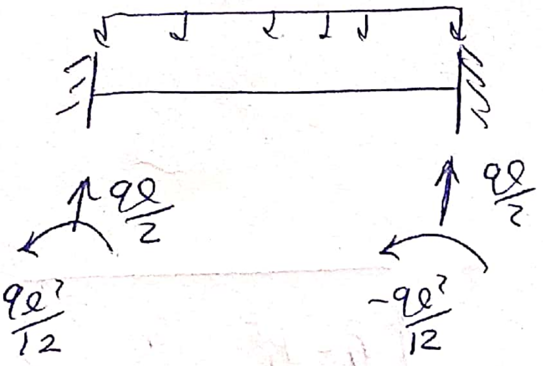
$$\longrightarrow \begin{Bmatrix} P_1 - P_1^F \\ M_1 - M_1^F \\ P_2 - P_2^F \\ M_2 - M_2^F \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ & K_{22} & K_{23} & K_{24} \\ & & K_{33} & K_{34} \\ & & & K_{44} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{Bmatrix}$$

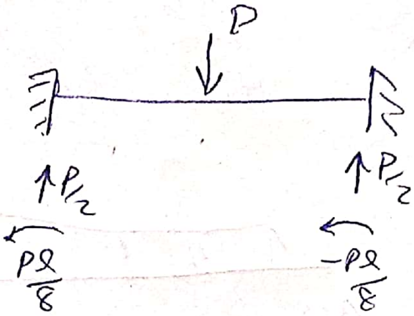
$$\begin{Bmatrix} P_1 \\ M_1 \\ P_2 \\ M_2 \end{Bmatrix} - \begin{Bmatrix} P_1^F \\ M_1^F \\ P_2^F \\ M_2^F \end{Bmatrix} = \begin{bmatrix} & \end{bmatrix} \begin{Bmatrix} \\ \end{Bmatrix}$$

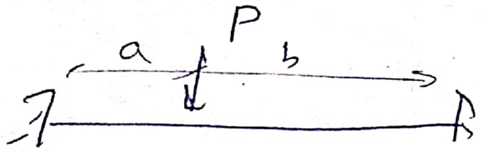
$$F - F^F = \underset{\sim}{K} \Delta$$

$$\underset{\sim}{F} = \underset{\sim}{K} \underset{-}{\Delta} + \underset{\sim}{F}^F$$

(نیروهای گیرداری)







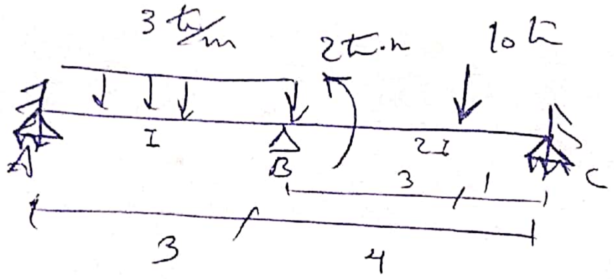
$R_1 = \frac{Pb}{l} + \frac{M_1+M_2}{l}$

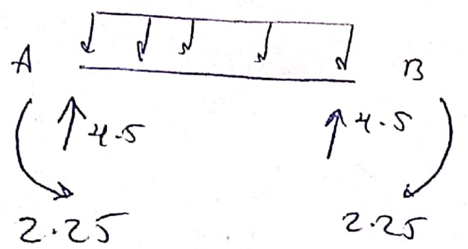
$M_1 = \frac{Pab^2}{l^2}$

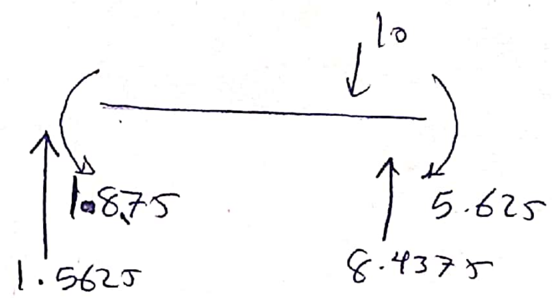
$M_2 = -\frac{Pa^2b}{l^2}$

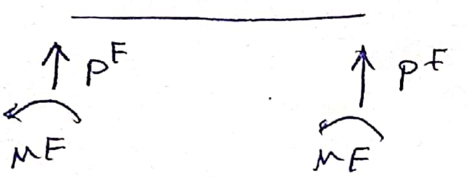
$R_2 = \frac{Pa}{l} - \frac{\left(\frac{Pab^2}{l^2} - \frac{Pa^2b}{l^2}\right)}{l}$

$= \frac{Pa}{l} - \frac{Pab}{l^3}(b-a)$

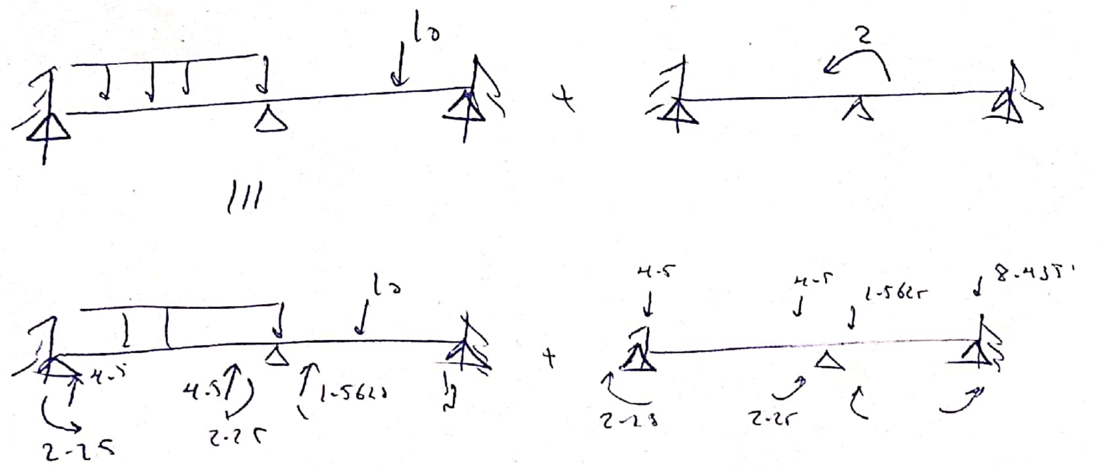








$$r_F^{(1)} = \begin{Bmatrix} 4.5 \\ 2.25 \\ 4.5 \\ -2.25 \end{Bmatrix} \qquad r_F^{(2)} = \begin{Bmatrix} 1.5625 \\ 1.875 \\ 8.4375 \\ -5.625 \end{Bmatrix}$$



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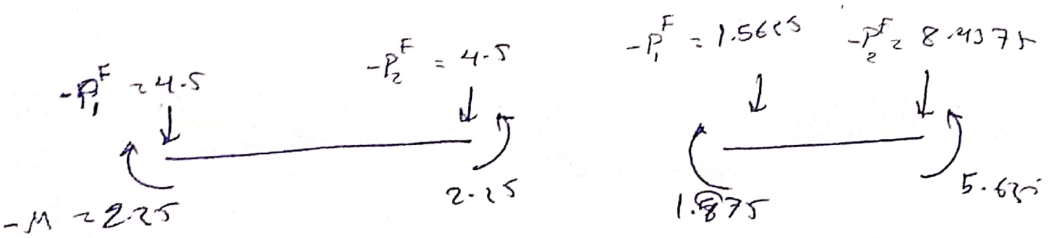
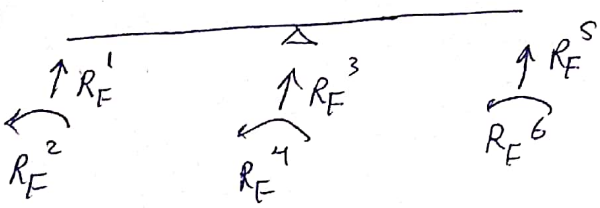


$r^{(1)} - r_F^{(1)} = K^{(1)} \Delta^{(1)}$ $\qquad$ $r^{(2)} - r_F^{(2)} = K^{(2)} \Delta^{(2)}$

$r^{(1)} = K^{(1)} \Delta^{(1)} + r_F^{(1)}$ $\qquad$ $r^{(2)} = K^{(2)} \Delta^{(2)} + r_F^{(2)}$

سیستم $\quad R = K\Delta + R_F$



$R_F^1 = P_1^{F(1)} = 4.5$

$R_F^2 = M_1^{F(1)}$

$R_F^3 = P_2^{F(1)} + P_1^{F(2)}$

$R_F^4 = M_2^{F(1)} + M_1^{F(2)}$

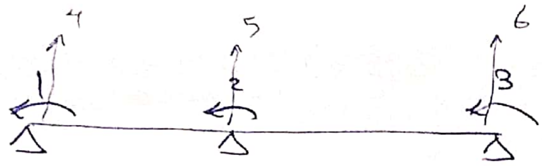
$R_F^5 = P_2^{F(2)}$

$R_F^6 = M_2^{F(2)}$

از جدول کد شیت ها کمک ←

	1	2	3	4
1	1	2	3	4
2	3	4	5	6

حل شکل ← به روش اعضا [illegible]



$$R = K\Delta + R_F$$

$$R = \begin{Bmatrix} M_1 = 0 \\ M_2 = 2 \\ M_3 = 0 \\ P_1 \\ P_2 \\ P_3 \end{Bmatrix} , \quad \Delta = \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \Delta_4 = 0 \\ \Delta_5 = 0 \\ \Delta_6 = 0 \end{Bmatrix}$$

$$R_F = \begin{Bmatrix} R_F^1 \\ R_F^2 \\ R_F^3 \\ \hline R_F^4 \\ R_F^5 \\ R_F^6 \end{Bmatrix}$$

	1	2	3	4
1	4	1	5	2
2	5	2	6	3

$$R_F^1 = R_{F_2}^{(1)} = 2.25$$

$$R_F^2 = -2.25 + 1.875 = -0.375$$

$$R_F^3 = -5.625 = -5.625$$

$$R_F^4 = 4.5$$

$$R_F^5 = 4.5 + 1.5625 = 6.0625$$

$$R_F^6 = 8.4375 =$$

$$\begin{Bmatrix} 0 \\ 2 \\ 0 \\ \hline P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \frac{EI}{72}\left[\begin{array}{ccc|ccc} 96 & 48 & 0 & 48 & -48 & 0 \\ & 240 & 72 & 48 & 6 & -54 \\ & & 144 & 0 & 54 & -54 \\ \hline & & & 32 & -32 & 0 \\ & & & & 59 & -27 \\ & & & & & 27 \end{array}\right] \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \hline 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 2.25 \\ -0.375 \\ -5.625 \\ \hline 4.5 \\ 6.0625 \\ 8.4370 \end{Bmatrix}$$

$$\begin{Bmatrix} F_u \\ F_c \end{Bmatrix} = \begin{bmatrix} \underset{\sim}{K_{uu}} & \underset{\sim}{K_{uc}} \\ \underset{\sim}{K_{cu}} & K_{cc} \end{bmatrix} \begin{Bmatrix} \underset{\sim}{\Delta_u} \\ \underset{\sim}{\Delta_c} = \underset{\sim}{0} \end{Bmatrix} + \begin{Bmatrix} R^F_u \\ \hline R^F_c \end{Bmatrix}$$

$$\begin{cases} F_u = \underset{\sim}{K_{uu}}\, \underset{\sim}{\Delta_u} + R^F_{\underset{\sim}{u}} \\ F_c = K_{cu}\, \Delta_u + R^F_c \end{cases} \longrightarrow \underset{\sim}{F_u} - R^f_u = \underset{\sim}{K_{uu}}\, \underset{\sim}{\Delta_u}$$

$$\underset{\sim}{\Delta_u} = \underset{\sim}{K_{uu}^{-1}}\,(\underset{\sim}{F_u} - R^F_u)$$

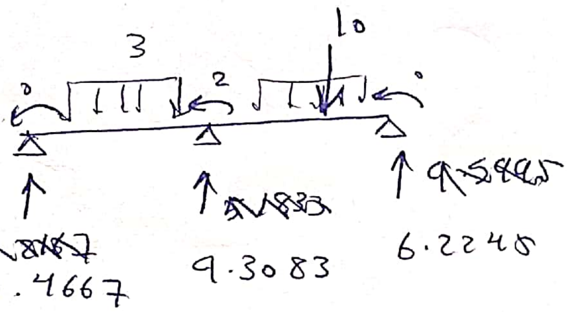
$$\begin{Bmatrix} 0 \\ 2 \\ 0 \end{Bmatrix} = \frac{EI}{72}\begin{bmatrix} 96 & 48 & 0 \\ & 240 & 72 \\ & & 144 \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} + \begin{Bmatrix} 2.25 \\ -0.375 \\ -5.625 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ 2 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 2.25 \\ -0.375 \\ -5.675 \end{Bmatrix} = \underset{\sim}{K_{uu}}\, \Delta_u$$

$$\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} -1.825 \\ 2.75 \\ +2.675 \end{Bmatrix} \times 1.^{-3}$$

$$F_c = K_{cu}\, \Delta_u + R^F_c = \frac{EI}{72}\begin{bmatrix} 48 & 48 & 0 \\ -48 & 6 & 54 \\ 0 & -54 & -54 \end{bmatrix} \times \begin{Bmatrix} -2.95 \\ 2.525 \\ -4.075 \end{Bmatrix} \times 1.^{-3} + \begin{Bmatrix} 4.5 \\ 6.0625 \\ 8.437 \end{Bmatrix}$$

$$\underset{\sim}{F_c} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} 3.4667 \\ 9.3083 \\ 6.2245 \end{Bmatrix}$$

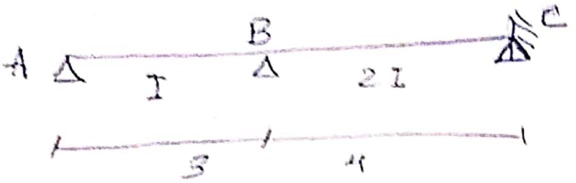


$\sum F_y = 0$ ✓

$\sum M = 0$ ✓

برای اعضا $\underset{\sim}{r}^{(1)} = k^{(1)} \Delta^{(1)} + r_f^{(1)}$ $\underset{\sim}{r}^{(2)} = k^{(2)} \Delta^{(2)} + r_f^{(2)}$

مطلوبست محاسبه نیروهای تکیه‌گاهی در [illegible]

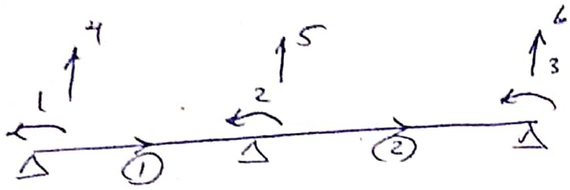


$\Delta_A = 2\ \text{cm} \downarrow$

$\Delta_B = 1.2\ \text{cm} \downarrow$

$\Delta_C = 1\ \text{cm} \downarrow$

حل



مجهولات $\quad \underset{\sim}{\Delta}_u = \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} \qquad \underset{\sim}{\Delta}_c = \begin{Bmatrix} \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{Bmatrix} = \begin{Bmatrix} -2 \\ -1.2 \\ -1 \end{Bmatrix} \times 10^{-2}\ (\text{m})$

$$\underset{\sim}{R}_u = \begin{Bmatrix} M_1 = 0 \\ M_2 = 0 \\ M_3 = 0 \end{Bmatrix} \quad ; \quad \underset{\sim}{R}_c = \begin{Bmatrix} P_4 \\ P_5 \\ P_6 \end{Bmatrix}$$

$$\begin{Bmatrix} \underset{\sim}{R}_u \\ \underset{\sim}{R}_c \end{Bmatrix} = \begin{bmatrix} \underset{\sim}{K}_{uu} & \underset{\sim}{K}_{uc} \\ \underset{\sim}{K}_{cu} & \underset{\sim}{K}_{cc} \end{bmatrix} \begin{Bmatrix} \underset{\sim}{\Delta}_u \\ \underset{\sim}{\Delta}_c \end{Bmatrix} + \begin{Bmatrix} \underset{\sim}{R}_u^F \\ \underset{\sim}{R}_c^F \end{Bmatrix}$$

$$\underset{\sim}{R}_u = \underset{\sim}{K}_{uu} \Delta_u + \underbrace{\underset{\sim}{K}_{uc} \underset{\sim}{\Delta}_c + \underset{\sim}{R}_u^F} \longrightarrow \underset{\sim}{R}_u - (\underset{\sim}{K}_{uc} \underset{\sim}{\Delta}_c + \underset{\sim}{R}_u^F) = K_{uu}\Delta_u$$

$$R_c = \underset{\sim}{K}_{cu} \underset{\sim}{\Delta}_u + K_{cc} \Delta_c + R_c^F$$

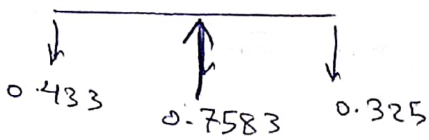
$$\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} - \left(\frac{1000}{72} \begin{bmatrix} 48 & -48 & 0 \\ 48 & 6 & -54 \\ 0 & 54 & -54 \end{bmatrix} \begin{Bmatrix} -2 \\ -1.2 \\ -1 \end{Bmatrix} \times 10^{-2} + \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \right) = \begin{Bmatrix} +5.33 \\ +6.83 \\ +1.5 \end{Bmatrix}$$

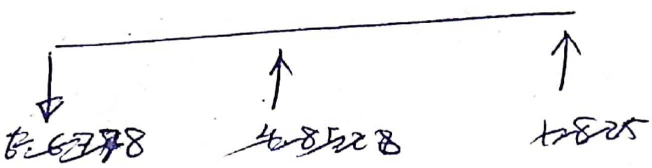
$$\underset{\sim}{\Delta_u} = \underset{\sim}{K_{uu}}^{-1} \left(R_u - \left(K_{uc} \Delta_c + \underset{\sim}{R^F_u} \right) \right) = \begin{Bmatrix} +3.317 \\ +1.367 \\ +0.0667 \end{Bmatrix} \times 10^{-3}$$

$$\underset{\sim}{R_c} = \underset{\sim}{K_{cu}} \underset{\sim}{\Delta_u} + K_{cc} \Delta_c + R^F_c =$$ Ixaxis +, J∧G

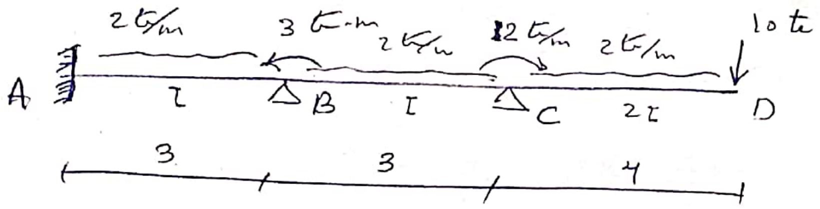
$$= \frac{1000}{72} \begin{bmatrix} 48 & 48 & 0 \\ -48 & 6 & 54 \\ 0 & -54 & -54 \end{bmatrix} \begin{Bmatrix} \\ \\ \end{Bmatrix} + \frac{1000}{72} \begin{bmatrix} 32 & -32 & 0 \\ & 59 & -27 \\ & & 27 \end{bmatrix} \begin{Bmatrix} \\ \\ \end{Bmatrix}$$

$$R_c = \begin{Bmatrix} \cancel{-6.678} \\ \cancel{4.8527} \\ \cancel{1.825} \end{Bmatrix} = \begin{Bmatrix} -0.433 \\ 0.7583 \\ -0.325 \end{Bmatrix}$$





تحلیل تیر با در نظر گرفتن نشست

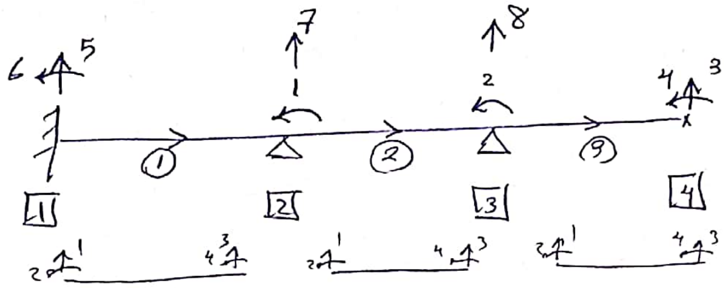


$$A\begin{cases}\Delta = 1^{mm}\downarrow \\ \theta = 10^{-2}\,rad\end{cases}$$

$\Delta_B = 1.5^{mm}\downarrow$

$\Delta_C = 2^{mm}\downarrow$

نشست



$$K^{(1)} = K^{(2)} = EI\begin{bmatrix} \frac{12}{27} & \frac{6}{9} & -\frac{12}{27} & \frac{6}{9} \\ & \frac{4}{3} & -\frac{6}{9} & \frac{2}{3} \\ & & \frac{12}{27} & -\frac{6}{9} \\ & & & \frac{4}{3}\end{bmatrix} = \frac{EI}{27}\begin{bmatrix} 12 & 18 & -12 & 18 \\ & 36 & -18 & 18 \\ & & 12 & -18 \\ & SYM & & 36\end{bmatrix}$$

$$K^{(3)} = \frac{EI}{8}\begin{bmatrix} 3 & 6 & -3 & 6 \\ & 16 & -6 & 8 \\ & & 3 & -6 \\ & & & 16\end{bmatrix}$$

	1	2	3	4
1	5	6	7	1
2	7	1	8	2
3	8	2	3	4

$K_{11} = K_{44}^{(1)} + K_{22}^{(2)} = \frac{36}{27} \times 2EI = \frac{72}{27}EI$

$K_{12} = K_{24}^{(2)} = \frac{18EI}{27}$

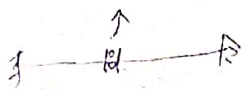
$K_{13} = K_{14} = 0$

$K_{15} = K_{41}^{(1)} = \frac{18EI}{27}$

$K_{16} = \frac{18EI}{27}$

$K_{17} = K_{43}^{(1)} + K_{21}^{(2)} = -\frac{18EI}{27} + \frac{18EI}{27} = 0$

$K_{18} = K_{23}^{(2)} = -\frac{18}{27}EI$



$$K_{22} = K_{44}^{(2)} + K_{22}^{(3)} = \frac{36EI}{27} + \frac{16EI}{8} = \frac{720EI}{216} = \frac{240}{72}EI$$

$$K_{23} = K_{23}^{(3)} = -\frac{6EI}{8}$$

$$K_{24} = K_{24}^{(3)} = \frac{8EI}{8}$$

$$K_{25} = K_{26} = 0$$

$$K_{27} = K_{4\cancel{7}}^{(2)} = +\frac{18EI}{27}$$

$$K_{28} = K_{43}^{(2)} + K_{21}^{(3)} = -\frac{18EI}{27} + \frac{6EI}{8} = \frac{18EI}{216} = \frac{6EI}{72}$$

$$K_{33} = K_{33}^{(3)} = \frac{3EI}{8}$$

$$K_{34} = K_{34}^{(3)} = -\frac{6EI}{8}$$

$$K_{35} = K_{36} = K_{37} = 0$$

$$K_{38} = K_{32}^{(3)} = -\frac{6EI}{8}$$

$$K_{44} = K_{44}^{(3)} = 2EI$$

$$K_{45} = K_{46} = K_{47} = 0$$

$$K_{48} = K_{43}^{(3)} = -\frac{6EI}{8}$$

$$K_{55} = K_{11}^{(1)} = \frac{12EI}{27}$$

$$K_{56} = K_{12}^{(1)} = \frac{18EI}{27}$$

$$K_{57} = K_{13}^{(1)} = -\frac{12EI}{27}$$

$$K_{58} = 0$$

$$K_{66} = K_{22}^{(1)} = \frac{36EI}{27}$$

$$K_{67} = K_{23}^{(1)} = -\frac{18EI}{27}$$

$$K_{68} = 0$$

$$K_{77} = K_{33}^{(1)} + K_{11}^{(2)} = \frac{24EI}{27}$$

$$K_{78} = K_{13}^{(2)} = +\frac{12EI}{27}$$

$$K_{88} = K_{33}^{(2)} + K_{11}^{(3)}$$

$$= \frac{12EI}{27} + \frac{3EI}{8} = \frac{59EI}{72}$$

$$K = EI\left[\begin{array}{cccc|cccc}
\frac{72}{27} & \frac{18}{27} & 0 & 0 & \frac{18}{27} & \frac{18}{27} & 0 & -\frac{18}{27} \\
 & \frac{240}{72} & -\frac{54}{72} & \frac{72}{72} & 0 & 0 & \frac{18}{27} & \frac{6}{72} \\
 & & \frac{27}{72} & -\frac{54}{72} & 0 & 0 & 0 & -\frac{54}{72} \\
 & & & \frac{144}{72} & 0 & 0 & 0 & -\frac{54}{72} \\
\hline
 & & & & \frac{12}{27} & \frac{18}{27} & -\frac{12}{27} & 0 \\
 & & & & & \frac{36}{27} & -\frac{18}{27} & 0 \\
 & & & & & & \frac{24}{27} & -\frac{12}{27} \\
 & & & & & & & \frac{59}{72}
\end{array}\right]$$

$$\underset{\sim}{R} = \underset{\sim}{K}\,\underset{\sim}{\Delta} + \underset{\sim}{R_F}$$

$$\underset{\sim}{K} = \left[\begin{array}{c|c} \underset{\sim}{K_{uu}} & \underset{\sim}{K_{uc}} \\ \hline \underset{\sim}{K_{cu}} & \underset{\sim}{K_{cc}} \end{array}\right]$$

$$\underset{\sim}{\Delta} = \left\{\begin{array}{c} \underset{\sim}{\Delta_u} \\ \hline \underset{\sim}{\Delta_c} \end{array}\right\} = \left\{\begin{array}{c} \theta_1 \\ \theta_2 \\ \Delta_3 \\ \theta_4 \\ \hline \Delta_5 \\ \theta_6 \\ \Delta_7 \\ \Delta_8 \end{array}\right\}$$

$$K_{uu} = \frac{EI}{72}\begin{bmatrix} 192 & 48 & 0 & 0 \\ & 240 & -54 & 72 \\ & & 27 & -54 \\ & & & 144 \end{bmatrix}$$

$$K_{uc} = \frac{EI}{72}\begin{bmatrix} 48 & 48 & 0 & -48 \\ 0 & 0 & 48 & 6 \\ 0 & 0 & 0 & -54 \\ 0 & 0 & 0 & -54 \end{bmatrix} = K_{cu}'$$

$$K_{cc} = \frac{EI}{72}\begin{bmatrix} 32 & 48 & -32 & 0 \\ & 96 & -48 & 0 \\ & & 64 & -32 \\ & & & 59 \end{bmatrix}$$

$$\begin{Bmatrix} R_u \\ \hline R_c \end{Bmatrix} = \begin{Bmatrix} M_1 \\ M_2 \\ R_3 \\ M_4 \\ \hline R_5 \\ M_6 \\ R_7 \\ R_8 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ \hline R_5 \\ R_6 \\ R_7 \\ R_8 \end{Bmatrix} = \begin{Bmatrix} 3 \\ -2 \\ -10 \\ 0 \\ \hline R_5 \\ R_6 \\ R_7 \\ R_8 \end{Bmatrix}$$

$$\tilde{R}^F_u = \begin{Bmatrix} 0 \\ 1.17 \\ 4 \\ -2.67 \end{Bmatrix} \qquad \tilde{R}^F_c = \begin{Bmatrix} 3 \\ 1.5 \\ 6 \\ 7 \end{Bmatrix}$$

$$\begin{Bmatrix} \tilde{R}_u \\ \tilde{R}_c \end{Bmatrix} = \begin{bmatrix} K_{uu} & K_{uc} \\ K_{cu} & K_{cc} \end{bmatrix} \begin{Bmatrix} \tilde{\Delta}_u \\ \tilde{\Delta}_c \end{Bmatrix} + \begin{Bmatrix} \tilde{R}^F_u \\ \tilde{R}^F_c \end{Bmatrix}$$

$$\tilde{R}_u = \tilde{K}_{uu} \tilde{\Delta}_u + \left(K_{uc} \tilde{\Delta}_c + \tilde{R}^F_u \right)$$

$$\tilde{R}_c = K_{cu} \Delta_u + K_{cc} \Delta_c + \tilde{R}^F_c$$

$$\tilde{R}_u - \left(\tilde{K}_{uc} \tilde{\Delta}_c + R^F_u \right) = \tilde{K}_{uu} \tilde{\Delta}_u$$

$$\begin{Bmatrix} 3 \\ -2 \\ -10 \\ 0 \end{Bmatrix} - \left(\frac{1000}{72} \begin{bmatrix} 48 & 48 & 0 & -48 \\ 0 & 0 & 48 & 6 \\ 0 & 0 & 0 & -54 \\ 0 & 0 & 0 & -54 \end{bmatrix} \begin{Bmatrix} -10^{-3} \\ 10^{-2} \\ -1.5 \times 10^{-3} \\ -2 \times 10^{-3} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1.17 \\ 4 \\ -2.67 \end{Bmatrix} \right)$$

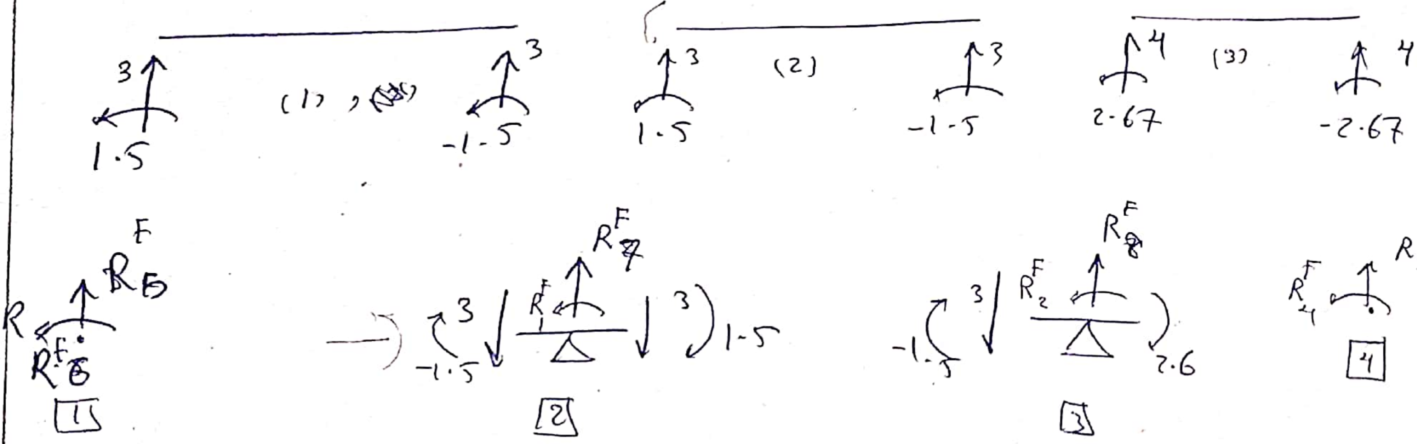
$$\begin{Bmatrix} -4.333 \\ -2.003 \\ -15.5 \\ 1.17 \end{Bmatrix} = \tilde{K}_{uu} \Delta_u$$

Scanned by CamScanner

$$\Delta = \left\{\begin{array}{c} \Delta_u \\ \hline \Delta_c \end{array}\right\} = \left\{\begin{array}{c} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \hline \Delta_5 \\ \Delta_6 \\ \Delta_7 \\ \Delta_8 \end{array}\right\} = \left\{\begin{array}{c} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \hline -10^{-3} \\ 10^{-2} \\ -1.5\times10^{-3} \\ -2\times10^{-3} \end{array}\right\}$$

نشست متقابل

نیروهای گیرداری



$R^F_1 = -1.5 + 1.5 = 0$

$R^F_2 = -1.5 + 2.67 = 1.17$

$R^F_3 = 4$

$R^F_4 = -2.67$

$$R_F = \left\{\begin{array}{c} R^F_u \\ \hline R^F_c \end{array}\right\} = \left\{\begin{array}{c} 0 \\ 1.17 \\ 4 \\ -2.67 \\ \hline 3 \\ 1.5 \\ 6 \\ 7 \end{array}\right\}$$

$R^F_5 = 3$ $\quad$ $R^F_7 = 6$

$R^F_6 = 1.5$ $\quad$ $R^F_8 = 3 + 4 = 7$

$$\underset{\sim}{\Delta u} = \begin{Bmatrix} 0.0116 \\ -0.053 \\ -0.372 \\ -0.112 \end{Bmatrix}$$